Lightning current and electromagnetic fields modeling

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Outline

• Physics of lightning flash development

• Lightning current modeling

• Reconstruction of lightning current from electromagnetic fields measurements
Introduction

Lightning is a transient, high-current electric discharge that has a total path length on the order of kilometers and occurs when enough charge builds up to produce electric fields that exceed the “breakdown field.” The main lightning activity is produced by thunderclouds and the majority of lightning flashes occur within the cloud and are called intra-cloud flashes. Instead cloud-to-ground flashes, less frequent than intra-cloud flashes, are the primary hazard for people and structures.
Thundercloud model

Electric dipole with a positively charged region above a negatively charged region.

A weaker, positively charged region at the base of the cloud makes it a double-dipole
A unidirectional, uniformly charged leader originating from a space charge source.

That model was called source charged model and, according to it, in the negative leader stage of a cloud to ground flash (CG), negative charge is progressively removed from the cloud and becomes distributed along the whole leader channel.

DOES NOT EXIST IN NATURE
A unidirectional, uniformly charged leader originating from a space charge source.

\[ \nabla^2 V = 0 \quad r > a \]

\[ \lim_{r \to \infty} \nabla V = E_0 \hat{e}_z \]

\[ V(a, \theta, \varphi) = 0 \]
A unidirectional, uniformly charged leader originating from a space charge source.

Original definition

\[ V(r, \theta) = E_0 \frac{a^3 - r^3}{r^2} \cos \theta \]

\[ \vec{E}(r, \theta) = E_0 \left( 2 \frac{a^3}{r^3} + 1 \right) \cos \theta \vec{e}_r + E_0 \left( \frac{a^3}{r^3} - 1 \right) \sin \theta \vec{e}_\theta \]

\[ Q = \int_{0}^{\frac{\pi}{2}} \int_{0}^{2\pi} \varepsilon_0 E_0 \cos \theta a^2 \sin \theta \, d\phi \, d\theta = \]

\[ = \pi \varepsilon_0 E_0 a^2 \int_{0}^{\frac{\pi}{2}} \sin 2\theta \, d\theta = 0 \]
Actual definition

The lightning process occurs as a bidirectional, bipolar, zero-net charge leader and electrodeless discharge

Corona discharge

Streamer

Leader
A neutral atom or molecule, in a region of strong electric field is ionized by a natural environmental event (for example, being struck by an ultraviolet photon or cosmic ray particle), to create a positive ion and a free electron.

The electric field accelerates these oppositely charged particles in opposite directions, separating them, preventing their recombination, and imparting kinetic energy to each of them.

The electron has a much higher charge/mass ratio and so is accelerated to a higher velocity than the positive ion. It gains enough energy from the field that when it strikes another atom it ionizes it, knocking out another electron, and creating another positive ion. These electrons are accelerated and collide with other atoms, creating further electron/positive-ion pairs, and these electrons collide with more atoms, in a chain reaction process called an electron avalanche.
Streamers vs. leaders

- Leader is a self-propagating discharge in an ambient electric field that produces a hot plasma channel with a continuing current.
- The leader polarity refers to the sign of the charge at its tip; so that a negative leader propagating downwards carries down a negative charge, while the whole net charge of the leader can be zero.
- Streamers are cold Corona filaments of the length of the few meters, while leaders are hot plasma channels and are self-propagating.
- The leader formation occurs during the transition from, or a transformation of, millions of corona streamers focused like sun rays on a magnifying glass on the tip of an existing conductive leader.
- The transition between streamers and leader is different between positive and negative leaders.
Types of lightning

- Negative cloud to ground: 90%.
- Positive cloud to ground: 5%.
- Negative ground to cloud: 3%.
- Positive ground to cloud: 2%.

Typically, a cloud to ground flash starts inside the cloud and so one can talk about a different process which is called intra cloud lightning event (IC).
Then, T4 shows of the current cut off in the Channel trunk connecting positive and negative parts of the tree.

The initial stage of a flash corresponds to the occurrence of a bipolar leader, with the negative leader on one end and a positive one on the other end of the bipolar lightning tree. The two leaders progress in opposite directions. Such stage of the process (from T1 to T3) is commonly called **stepped leader**.

T5 shows the progression of the positive leader, with intermittent occurrence of negative recoil leaders, while there is **no radiation sources at the negative end of the tree**.
CG negative flashes

• The cloud-to-ground flash has an initial development which is equal to the IC one. The most significant difference can be seen when that stepped leader touches ground.
• When the stepped leader gets near the ground (about 100 m), positive streamers move from the ground up toward the stepped leader. The streamers may come from almost any pointed object on the ground (trees, antennas, poles….)
• One of the streamers will meet the stepped leader, not necessarily the one from the tallest object. When they meet, a pulse of energy flows up toward the cloud (along the ionized path) and towards the ground. There is a rapid transfer of charge from cloud to ground.

This luminous pulse of electrical energy is called the return stroke
In t5 the current cut off in the negative leader and the progression of the positive one, with the intermittent happening of negative recoil leaders. When one of them touches again the ground, a subsequent stroke initiates and the cycle continues.
The same sequence happens until the positive leader touches ground. Then, the return stroke energizes the upward negative leader and determines the channel cut off. However, no new negative recoil leaders appear, which means that in positive cloud to ground flashes there is only one return stroke.
Upward flashes

• Less frequent and less understood


Upward flashes triggering mechanisms

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Lightning current modeling

- Lightning Overvoltages
- Overhead power lines
- Buried cables
- Telecommunication lines

Indirect vs. Direct
Lightning current modeling

Indirect

- Field to line coupling
- Lightning e.m. fields
- Modeling of the lightning current
Lightning current modeling

- **Gas or physical models**
  Solution of 3 hydrodynamic equations (also thermodynamic parameters).

- **Electromagnetic models**
  Lightning channel as thin wire antenna. Maxwell’s equations solved with numerical techniques (MOM).

- **Distributed circuit models**

- **Engineering models**
Lightning current modeling

Focus on:

- The main available engineering models:
  - The height dependent attenuation function $P$

- The available strategies to validate the proposed models:
  - Direct vs. Inverse Methods
Engineering models

- The current in the lightning channel at any time \( t \) and height \( z' \) is related to the base current by means of a suitable expression.

- Deliberately neglect the physics of the lightning return stroke, focus their attention on the agreement between model predicted electromagnetic fields and measurements.

- Two categories:
  - Transmission line type models
  - Traveling current source type models
Transmission line type models

The current in the lightning channel \( i(z',t) \) is related to the CURRENT SOURCE \( I_0 \) PLACED AT THE CHANNEL BASE by means of an height dependent attenuation function \( P(z') \).

\[
i(z',t) = I_0(0,t-z'/v) \ P(z')u(t-z'/v_f)
\]

\( v \): current wave propagation speed
\( v_f \): upward propagating front speed
Transmission line type models

Different expressions for $P$

Classical:
- **TL** \( P(z') = 1 \)
- **MTLL** \( P(z') = 1 - z'/H \)
- **MTLE** \( P(z') = \exp(-z'/\lambda) \)

More recent:
- **MTLT** \( P(z') = 0.5 \left[ 1 + \left( 1 - 2 \frac{z'}{H} \right)^3 \right] \)
- **MTLTS** \( P(z') = 0.25 \left[ 1 + \left( 1 - 2 \frac{z'}{H} \right)^3 \right]^2 \)
- **MTLTCOS** \( P(z') = 0.95 - 0.95 \frac{z'}{H} + 0.05 \cos \left( 5\pi \frac{z'}{H} \right) \)
Transmission line type models

MTLL

TL

MTLE
Transmission line type models

MTLTCOS

MTLT

MTLTS
Traveling current source models

- The return stroke current is generated at the upward moving return stroke front and propagates downward

- Bruce and Gold

- TCS model (Heidler)

- Diendorfer and Uman (DU) model
Traveling current source models

- Bruce and Gold

\[ I(z', t) = I_0(t) u \left( t - \frac{z'}{v_f} \right) \]
Traveling current source models

- **TCS**

\[ I(z', t) = I_0 \left( t + \frac{z'}{c} \right) u \left( t - \frac{z'}{v_f} \right) \]
Traveling current source models

**DU**

\[
I(z', t) = \left[ I_0 \left( t + \frac{z'}{c} \right) - I_0 \left( \frac{z'}{v^*} \right) e^{-\frac{t - \frac{z'}{v_f}}{\tau_D}} \right] u \left( t - \frac{z'}{v_f} \right)
\]

\[
v^* = \frac{c v_f}{c + v_f}, \quad \tau_D > 0
\]
Channel base current

- DEXP

\[ I_0(t) = I_m \left( e^{-\alpha t} - e^{-\beta t} \right) \]

- HEIDLER FORMULA

\[ I_0(t) = \frac{I_s}{\eta} \frac{(t / \tau_1)^n}{1 + (t / \tau_1)^n} \exp(-t / \tau_2) \]

- PRONY SERIES (recursive convolution integral)

- NCBC FUNCTIONS (analytical integral and FT and peak collocation)
Validation of the engineering models: *direct procedure*

- Consists of a 3-step procedure.

  (1) Choice of a return stroke current model and calculation of the electromagnetic field;

  (2) Comparison with measured field;

  (3) Modification of the model parameters for a good fitting.
Validation of the engineering models: *direct procedure*

- Main fields characteristics (Nucci et al. 1990)
  1. a sharp initial peak that varies approximately as the inverse distance beyond a kilometer or so in both electric and magnetic fields
  2. a slow ramp following the initial peak lasting less than 100 μs for electric fields measured within a few tens of kilometers
  3. a hump following the initial peak in magnetic fields within a few tens of kilometers
  4. a zero crossing within tens of milliseconds after the initial peak in both electric and magnetic field at 50 to 200 km
Validation of the engineering models: *direct procedure*

From: (V. Javor 2016)

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Inverse procedure (Delfino et al. 2002)

- **Measurement** of the electromagnetic field;

- **Solution of the integral equation** expressing the link between electromagnetic field and current;

- **Mathematical evaluation** of the return stroke current.
The problem

- Lightning channel as a vertical antenna above a perfectly conducting plane;

- Return stroke wavefront starts traveling up at $t = 0$.

\[
\begin{align*}
E_r(r, z, \omega) &= \frac{1}{4\pi\varepsilon_0} \int_{-H}^{H} G_r(r, z, z', \omega) \exp\left[-j\omega \frac{R}{c}\right] \hat{I}(z', \omega) dz' \\
E_z(r, z, \omega) &= \frac{1}{4\pi\varepsilon_0} \int_{-H}^{H} G_z(r, z, z', \omega) \exp\left[-j\omega \frac{R}{c}\right] \hat{I}(z', \omega) dz' \\
H_\phi(r, z, \omega) &= \frac{1}{4\pi} \int_{-H}^{H} G_\phi(r, z, z', \omega) \exp\left[-j\omega \frac{R}{c}\right] \hat{I}(z', \omega) dz'
\end{align*}
\]
The problem (continued)

- The return stroke current spectrum is expressed by:
  \[ \hat{I}(z', \omega) = \hat{I}_0(\omega) \exp \left( -j\omega \frac{|z'|}{v} \right) P(z') \]

- At ground level, one has:
  \[ E_z(r, 0, \omega) = \frac{I_0(\omega)}{4\pi\varepsilon_0} \int_{-H}^{H} G_z(r, 0, z', \omega) \exp \left[ -j\omega \left( \frac{R}{c} + \frac{|z'|}{v} \right) \right] P(z') \, dz' \]
  \[ H_\phi(r, 0, \omega) = \frac{I_0(\omega)}{4\pi} \int_{-H}^{H} G_\phi(r, 0, z', \omega) \exp \left[ -j\omega \left( \frac{R}{c} + \frac{|z'|}{v} \right) \right] P(z') \, dz' \]

Fredholm integral equations of the first kind
The problem (continued)

Let

\[ D:= [r_1, r_2] \times [\omega_1, \omega_2] \]

\[ A_z : L^2_\mathbb{R}([-H, H]) \ni P \mapsto \int_{-H}^H K_z(r, z, \omega, z') P(z') \, dz' \in L^2_\mathbb{C}(D) \]

\[ K_z(r, z, \omega, z') = \frac{1}{4\pi\varepsilon_0} G_z(r, z, \omega, z') \exp\left(-j \omega \left(\frac{R}{c} + \frac{|z'|}{v}\right)\right) \]

and

\[ A_\phi : L^2_\mathbb{R}([-H, H]) \ni P \mapsto \int_{-H}^H K_\phi(r, z, \omega, z') P(z') \, dz' \in L^2_\mathbb{C}(D) \]

\[ K_\phi(r, z, \omega, z') = \frac{1}{4\pi} G_\phi(r, z, \omega, z') \exp\left(-j \omega \left(\frac{R}{c} + \frac{|z'|}{v}\right)\right) \]
The problem (continued)

Operatorial form

\[
\hat{I}_0 A_\phi (P) = H_\phi (\cdot, 0, \cdot)
\]

\[
\hat{I}_0 A_z (P) = E_z (\cdot, 0, \cdot)
\]

Solution procedure (Delfino et al. 2002)

Let:

\[
B = \left\{ \varphi_n \in L^2_\mathbb{R} \left( [-H, H] \right) : n = 1, 2, \ldots \right\}
\]

\[
P(z') = \sum_{n=1}^{\infty} p_n \varphi_n (z') \quad \forall z' \in [-H, H]
\]
The problem (continued)

Expanding $P$ with the basis $B$, the problem becomes

$$\sum_{n=1}^{N} p_n \left[ A_z (\varphi_n) \right] (r, \omega) - \lambda(\omega) E_z (r, 0, \omega) = 0$$

$$\sum_{n=1}^{N} p_n \left[ A_\phi (\varphi_n) \right] (r, \omega) - \lambda(\omega) H_\phi (r, 0, \omega) = 0$$

where

$$\lambda(\omega) = \frac{1}{\hat{I}_0(\omega)}$$

The coefficients are the unknown
The problem (continued)

Choice of the basis

(Delfino et al. 2002)

\[ \varphi_n(z') = T_{n-1}\left(\frac{|z'|}{H}\right) = \cos\left((n-1)\arccos\left(\frac{|z'|}{H}\right)\right) \]

(Andreotti et a. 2006)

\[ \varphi_n(z') = \left(1 - \left(\frac{|z'|}{H}\right)^2\right)^{s-1/2} C_n^s \left(\frac{|z'|}{H}\right) \]

Accounts for the current behaviour at edges
The problem (continued)

Distance-based identification

Sample one of the fields at one frequency and in N locations $r_s$
The problem (continued)

Distance-based identification

\[ \sum_{n=1}^{N} p_n \left[ A_z \left( \varphi_n \right) \right] (r_s, \omega) - \lambda(\omega) E_z (r_s, 0, \omega) = 0 \]

\[ \sum_{n=1}^{N} p_n \varphi_n (0) = 1 \]

N+1 eq. In N+1 unknowns: linear algebraic system
The problem (continued)

Frequency-based identification

Sample one of the fields at two distances $r_i$ and in $N-1$ frequencies $\omega_s$
The problem (continued)

Frequency-based identification

\[
\sum_{n=1}^{N} p_n \left[ A_z(\varphi_n) \right](r_i) - \lambda(\omega_s) E_z(r_i, 0, \omega_s) = 0
\]

\[
\sum_{n=1}^{N} p_n \left[ A_\phi(\varphi_n) \right](r_i) - \lambda(\omega_s) H_\phi(r_i, 0, \omega_s) = 0
\]

\[
\sum_{n=1}^{N} p_n \varphi_n(0) = 1
\]

\[
M \begin{bmatrix} p_1 \\ \vdots \\ p_N \\ \lambda(\omega_1) \\ \vdots \\ \lambda(\omega_{N-1}) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}
\]

2N-1 eq. In 2N-1 unknowns: linear algebraic system
Validation on Engineering models

- **MTLE reconstruction** (H = 7 km) in the frequency range 50 Hz ÷ 1 MHz with N = 12 expansion functions.
A bit on integral equations

- A first kind Fredholm integral equation

1. Given the linear operator $A$

$$A : X \to Y$$

$$(Af)(y) = \int_{a}^{b} K(x, y) f(x) dx$$

2. Given the integral equation

$$g(y) = \int_{a}^{b} K(x, y) f(x) dx$$

or, equivalently

$$g = Af$$

where $K(x,y)$ is the kernel, $f(x)$ is the unknown and $g(y)$ is the free term.

- What’s the problem with it?
A bit on integral equations

Often the solution is obtained discretizing the problem:

\[ g(y) = \int_{a}^{b} K(x, y) f(x) dx \]

↓ quadrature rule

\[ g(y) = \sum_{j=1}^{n} K(x_j, y) f(x_j) \]

↓ collocation

\[ g(y_i) = \sum_{j=1}^{n} K(x_j, y_i) f(x_j) \rightarrow \text{algebraic linear system} \]
A bit on integral equations

Example (from Fox and Goodwin)

\[
\left[ \frac{(1 + s^2)^{\frac{3}{2}} - s^2}{3} \right] = \int_{0}^{1} \left( t^2 + s^2 \right)^{\frac{1}{2}} f(t) dt
\]

whose solution is \( f(t) = t \)

Applying the discretization method one has:

\[
\left[ \frac{(1 + t_i^2)^{\frac{3}{2}} - t_i^2}{3} \right] = \sum_{j=1}^{n} \left( t_j^2 + t_i^2 \right)^{\frac{1}{2}} x_j
\]
A bit on integral equations

being

\[ t_j = \frac{(2j - 1)}{2n} \]

Exact

Discretized

n=20

[Graph showing a comparison between exact and discretized solutions for integral equations]
A bit on integral equations

- The problem is ill-posed: the simple round-off error in the computation of the free term makes the procedure ineffective.

- A problem is well-posed if its solution exists, is unique and depends continuously on the data. If one of these conditions is not satisfied, it is ill-posed. Typically the third condition misses!!

- It can be shown that, when a linear operator $A$ is compact, the associated problem $Ax=b$ is ill-posed.
Our problem

- Recalling that

\[
\hat{I}_0 A_{\phi} (P) = H_{\phi} (\cdot, 0, \cdot)
\]

\[
\hat{I}_0 A_{z} (P) = E_{z} (\cdot, 0, \cdot)
\]

where

\[
A_{z} : L^2_{\mathbb{R}}([-H, H]) \ni P \mapsto \int_{-H}^{H} K_{z}(r, z, \omega, z') P(z') \, dz' \in L^2_{\mathbb{C}}(D)
\]

\[
A_{\phi} : L^2_{\mathbb{R}}([-H, H]) \ni P \mapsto \int_{-H}^{H} K_{\phi}(r, z, \omega, z') P(z') \, dz' \in L^2_{\mathbb{C}}(D)
\]

\[
D := [r_1, r_2] \times [\omega_1, \omega_2]
\]

\[
K_{z}(r, z, \omega, z') = \frac{1}{4\pi\varepsilon_0} G_{z}(r, z, \omega, z') e^{-j\omega \left( \frac{R}{c} + \frac{|z'|}{v} \right)}
\]

\[
K_{\phi}(r, z, \omega, z') = \frac{1}{4\pi} G_{\phi}(r, z, \omega, z') e^{-j\omega \left( \frac{R}{c} + \frac{|z'|}{v} \right)}
\]
Our Problem

- Now, as

\[
\int_{D \times [-H, H]} \left| K_{z(\phi)}(r, 0, z', \omega) \right|^2 \, dr \, d\omega \, dz' < \infty
\]

Both operators are compact \( \rightarrow \) ILL-POSED PROBLEM

Almost all inverse problems coming from realistic physical situations are \textbf{ill-posed}.

A theoretical available basis can provide extremely large number of expansion functions or even unfruitful results in the reconstruction process.
Solving ill-posed problems

- Given the ill-posed problem \( g = Af \)

The solution existence issue is faced looking for one of the pseudo solutions \( u \), i.e.:

\[
u \text{ s.t. } \|Au - g\| = \min
\]

and, among them, evaluating the generalized solution, \( f^\dagger \) i.e. the pseudosolution with minimum norm

\[
f^\dagger = \sum_{k=1}^{\infty} \frac{1}{\sigma_k} (g, v_k) u_k
\]

Eigenvalues of \( A^*A \)

Eigenfunctions of \( A^*A \)

Eigenfunctions of \( AA^* \)
But, as

\[
\frac{\|\delta f\|}{\|f\|} \leq \|A\|\|A^+\| \frac{\|\delta g\|}{\|g\|}
\]

An experimental error $\delta g$ can be amplified in an unacceptable way

REGULARIZATION THEORY: solve another problem, but well-posed
Solving ill-posed problems

The one-parameter family of operators:

\[ R_\alpha : Y \to X \]

\[ R_\alpha \text{ bounded} \]

\[ \lim_{\alpha \to 0} \left\| R_\alpha g - f^\dagger \right\| = 0 \]

is called regularization algorithm.

If \( g \) is affected by errors, i.e.

\[ g_\delta = Af^\dagger + \omega_\delta \]

\[ \| \omega_\delta \| = \delta \]
Solving ill-posed problems

\[ \| R_\alpha g_\delta - f^\dagger \| \leq \| R_\alpha Af - f^\dagger \| + \delta \| R_\alpha \| \]

Approximation error that goes to zero if \( \alpha \) goes to zero

Noise on data error that increases if \( \alpha \) goes to zero

Optimal value of \( \alpha \) to be found as a function of \( \delta \)
Solving ill-posed problems:

Basic regularization algorithms

1. Truncated Singular Value Decomposition (TSVD)
2. Tikhonov method
3. Landweber method

Criteria to choose $\alpha$

1. Discrepancy principle
2. L-curve method
Solving ill-posed problems:

Basic regularization algorithms

1. **Truncated Singular Value Decomposition (TSVD)**

Parameter to be chosen

\[
R_{1g} = \sum_{k=1}^{N} \frac{1}{\sigma_k} (g, v_k) u_k
\]

Easiest method: truncation of the generalized solution formula
Solving ill-posed problems:

Basic regularization algorithms

2. Tikhonov method: solve a «similar» problem

\[
\left( A^* A + \alpha I \right) f_\alpha = A^* g
\]

Parameter to be chosen

Unique solution with continuous dependence on data

\[
f_\alpha = R_\alpha g = \sum_{k=1}^{\infty} \frac{\sigma_k}{\sigma_k^2 + \alpha} \left( g, v_k \right) u_k
\]
Solving ill-posed problems:

Basic regularization algorithms

3. Landweber method: solve a «similar» problem

\[ f_{n+1} = f_n + \tau \left( A^* g - A^* A f_n \right) \]

\[ f_0 = 0 \quad \tau \leq \frac{2}{\|A\|^2} \]

Parameter to be chosen

The sequence \( f_n \) converges to the generalized solution
Solving ill-posed problems:

Criteria to choose $\alpha$

1. Discrepancy principle: solve

$$\|Af_\alpha - g_\delta\| = \delta$$

2. L-curve method
At present, TSVD and Tikhonov methods have been applied by (Andreotti et al. 2012 and 2013), but without experimental data.
Conclusions

- Physics of lightning flash development
- Analysis of different engineering models
- Their typical validation
- Inverse procedure to identify the return stroke current
- The troubles with inverse problems
- Regularization techniques

Perspectives

■ The possibility of applying the different regularization techniques on a set of different fields measurements should be investigated.